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4D, $\mathcal{N} = 1$ higher spin gauge superfields and quantized twistors

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Abstract: For the gauge massless higher spin 4D, $\mathcal{N} = 1$ off-shell supermultiplets previously developed, we provide evidence of a twistor-like oscillator realization that is intrinsically related to the superfield structure of the dynamical variables and gauge transformations. Gauge invariant field strengths and linearized Bianchi identities for these multiplets are worked out. It is further argued, inspired by earlier non-supersymmetric constructions due to Klishevich and Zinoviev, that a massive superspin-s multiplet can be described as a gauge-invariant dynamical system involving massless multiplets of superspins $s, s - 1/2, \ldots, 0$. A gauge-invariant formulation for the massive gravitino multiplet is discussed in some detail.

Keywords: Supersymmetric Effective Theories, BRST Symmetry.

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1. Introduction

In four space-time dimensions, lagrangian formulations for massive fields of arbitrary spin were constructed thirty years ago [1], as a partial realization of the Fierz-Pauli program [2]. A few years later, the Singh-Hagen models [1] were used to derive lagrangian formulations for gauge massless fields of arbitrary spin [3]. The massless construction of [3] was then extended to (anti) de Sitter space [4], as well it stimulated the appearance of elegant reformulations and generalizations, see e. g. [5, 6].

In supersymmetric field theory, the supersymmetric analogue of the Casimir operator spin is called the *superspin* [7 − 9] (similarly, there exists natural supersymmetric extensions of the helicity [10 − 12]). It is therefore of some interest to develop supersymmetric extensions of the models discovered in [1, 3]. As was first demonstrated in the work of [13] and later in [6], on-shell massless multiplets of arbitrary superspin are easily obtained by putting together two massless spin-s and spin-$(s + 1/2)$ actions, derived in [3] or [6], and then guessing the structure of supersymmetry transformations. In such an approach, however, the supersymmetry transformations form a closed algebra only on the mass shell. It proves to be more difficult to construct off-shell massless higher superspin multiplets. The latter problem was solved in [14, 15] (see [11] for a review) building on the prepotential structure of $\mathcal{N}=1$ superfield supergravity [16] (see [10, 11] for reviews) as a guiding principle. For each superspin $s > 3/2$, these publications provide two dually equivalent off-shell realizations in 4D, $\mathcal{N}=1$ superspace. At the component level, each of the two superspin-s actions [14, 15] reduces, *upon* imposing a Wess-Zumino-type gauge and eliminating the auxiliary fields, to a sum of the spin-s and spin-$(s+1/2)$ actions [3]. On the mass shell, the only independent gauge-invariant field strengths in these models are exactly the higher spin on-shell field strengths first identified in "Superspace" [10]. The gauge massless higher spin supermultiplets of [14, 15] were also generalized to $\mathcal{N}=1$ anti-de Sitter superspace [17]. In addition, there have been developed 4D, $\mathcal{N}=2$ *off-shell* massless higher spin supermultiplets [18] (see also [19]), as well as a generating superfield action for arbitrary superspin massless multiplets in 4D, $\mathcal{N}=1$ anti-de Sitter superspace [20].

In the massive case, higher spin supermultiplets have never been constructed beyond superspin-3/2;[1] only the cases of massive gravitino multiplet (superspin-1) and massive graviton multiplet (superspin-3/2) have been studied in some detail [21 − 29], both at the off-shell and on-shell levels. For constructing massive higher spin supermultiplets, one could try to develop a direct extension of the Singh-Hagen approach [1]. However it seems less formidable, and also conceptually very appealing, to look for a supersymmetric extension of the approach advocated a few years ago by Klishevich and Zinoviev [30, 31]. In their approach, the massive spin-s particle is described by a gauge-invariant action which involves all the massless fields [3] of spins $s, s-1, \dots$, and possesses the properties: (i) in the massless limit, the action becomes a sum of the massless actions [3] of spins $s, s-1, \dots$; (ii) the gauge symmetry is a mass-dependent deformation of the massless gauge transformations; (iii) the gauge freedom can be used to choose a Wess-Zumino-type gauge condition in which the action reduces to the massive spin-s action of [1]. In a sense, the scheme developed in [30, 31] is a higher spin analogue of the Stückelberg construction. In the supersymmetric case, a gauge-invariant realization for the massive superspin-s multiplet should involve massless multiplets of superspins $s, s-1/2, \dots, 0$.

Assuming the existence of a gauge-invariant formulation for massive higher superspin multiplets, the gauge massless models introduced in [14, 15] should clearly form a natural starting point. But for each massless superspin $s > 3/2$, there are two dually equivalent realizations (there exist three off-shell realizations for the massless multiplets of superspin $s=0$ and 1, and four off-shell realizations for the massless superspin-3/2 multiplet). It is not clear a priori which one should occur as a building block in the construction of massive supermultiplets[2] (probably, several dually equivalent formulations also exist for massive higher superspin mulptiplets, as in the case of the massive vector multiplet (superspin-1/2) described in appendix A.2). We are not able to definitively answer this and similar questions currently. We believe that there still remain some important properties of the massless higher superspin multiplets which have to be studied beforehand. In addition, gauge-invariant descriptions for the massive gravitino multiplet (superspin-1) and massive

[1] As is well known [7, 8], a massive $\mathcal{N}=1$ multiplet of superspin s describes four propagating fields with the same mass but different spins $(s-1/2, s, s, s+1/2)$, see [10, 11] for reviews.

[2] Massless $\mathcal{N}=2$ supermultiplets are easier to construct [18, 20] using the transverse formulation for half-integer superspins [14] and the longitudinal formulation for integer superspins.

graviton multiplet (superspin-3/2) should be studied in detail (to the best of our knowledge, the observation given in [24] regarding the massive gravitino multiplet is the only result available).

This paper is organized as follows. In section 2 we review, following [14, 15], the superfield structure of massless higher superspin multiplets and their gauge symmetries. Section 3 is devoted to a twistor-like oscillator realization that is intrinsically related to the superfield structure of the dynamical variables and gauge transformations introduced. Using the quantized twistor, four BRST-like operators are defined. The compatibility of introducing the twistor-like oscillator realization is also discussed for the usual 4D, $\mathcal{N}=1$ abelian gauge theory. In sections 4 and 5, the structure of gauge-invariant field strengths and corresponding Bianchi identities, which occur in the massless higher superspin models is reviewed. In appendix A we collect the gauge-invariant action for the massless higher superspin multiplets [14, 15]. Three dually equivalent gauge-invariant realizations for the massive gravitino multiplet are discussed in appendix A.2. Finally, a gauge-invariant formulation for the massive gravitino multiplet is given in appendix B.

2. Higher spin gauge superfields

The off-shell gauge formulations [14, 15] for higher superspin massless multiplets in 4D, $\mathcal{N}=1$ Minkowski superspace[3] involve so-called transverse and longitudinal linear superfields, both as dynamical variables and gauge parameters. A complex tensor superfield $\Gamma_{\alpha(k)\dot\alpha(l)}$ subject to the constraint

$$\bar D^{\dot\beta}\,\Gamma_{\alpha(k)\dot\beta\dot\alpha(l-1)} = 0\,, \qquad l>0\,, \tag{2.1}$$

is said to be transverse linear. A longitudinal linear superfield $G_{\alpha(k)\dot\alpha(l)}$ is defined to satisfy the constraint

$$\bar D_{(\dot\beta}\,G_{\alpha(k)\dot\alpha_1\dots\dot\alpha_l)} = 0\,. \tag{2.2}$$

The above constraints imply that $\Gamma_{\alpha(k)\dot\alpha(l)}$ and $G_{\alpha(k)\dot\alpha(l)}$ are linear in the usual sense

$$\bar D^2\,\Gamma_{\alpha(k)\dot\alpha(l)} = \bar D^2\,G_{\alpha(k)\dot\alpha(l)} = 0\,. \tag{2.3}$$

In the case $l=0$, constraint (2.1) should be replaced by $\bar D^2\Gamma_{\alpha(k)}=0$. Constraint (2.2) for $l=0$ simply means that $G_{\alpha(k)}$ is chiral, $\bar D_{\dot\beta}\,G_{\alpha(k)}=0$. The constraints (2.1) and (2.2) can be solved in terms of unconstrained potentials $\Phi_{\alpha(k)\dot\alpha(l+1)}$ and $\Psi_{\alpha(k)\dot\alpha(l-1)}$ as follows:

$$\Gamma_{\alpha(k)\dot\alpha(l)} = \bar D^{\dot\beta}\Phi_{\alpha(k)\,\dot\beta\dot\alpha(l)}\,, \qquad G_{\alpha(k)\dot\alpha(l)} = \bar D_{(\dot\alpha_l}\Psi_{\alpha(k)\,\dot\alpha_1\cdots\dot\alpha_{l-1})}\,. \tag{2.4}$$

[3]Our superspace notation and conventions correspond to [11], in particular the flat superspace covariant derivatives are $D_A=(\partial_a, D_\alpha, \bar D^{\dot\alpha})$. Throughout this paper we consider only Lorentz tensors symmetric in their undotted indices and separately in their dotted ones. For a tensor of type (k,l) with k undotted and l dotted indices we use the shorthand notations $\Psi_{\alpha(k)\dot\alpha(l)} \equiv \Psi_{\alpha_1\dots\alpha_k\dot\alpha_1\dots\dot\alpha_l} = \Psi_{(\alpha_1\dots\alpha_k)(\dot\alpha_1\dots\dot\alpha_l)}$. Quite often we assume that the upper or lower indices, which are denoted by one and the same letter, should be symmetrized, for instance $\phi_{\alpha(k)}\psi_{\alpha(l)} \equiv \phi_{(\alpha_1\dots\alpha_k}\psi_{\alpha_{k+1}\dots\alpha_{k+l})}$. Given two tensors of the same type, their contraction is denoted by $f\cdot g \equiv f^{\alpha(k)\dot\alpha(l)}\,g_{\alpha(k)\dot\alpha(l)} = f^{\alpha_1\dots\alpha_k\dot\alpha_1\dots\dot\alpha_l}\,g_{\alpha_1\dots\alpha_k\dot\alpha_1\dots\dot\alpha_l}$.

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Two formulations for the massless multiplet of a half-integer superspin $s+1/2$ (with $s=1,2\ldots$) which were called in ref. [14] transverse and longitudinal, contain the following dynamical variables respectively:

$$\mathcal{V}^{\perp}_{s+1/2} = \Big\{ H_{\alpha(s)\dot\alpha(s)}\,,\ \Gamma_{\alpha(s-1)\dot\alpha(s-1)}\,,\ \bar\Gamma_{\alpha(s-1)\dot\alpha(s-1)} \Big\}\,, \tag{2.5}$$

$$\mathcal{V}^{\parallel}_{s+1/2} = \Big\{ H_{\alpha(s)\dot\alpha(s)}\,,\ G_{\alpha(s-1)\dot\alpha(s-1)}\,,\ \bar G_{\alpha(s-1)\dot\alpha(s-1)} \Big\}\,. \tag{2.6}$$

Here $H_{\alpha(s)\dot\alpha(s)}$ is real, $\Gamma_{\alpha(s-1)\dot\alpha(s-1)}$ transverse linear and $G_{\alpha(s-1)\dot\alpha(s-1)}$ longitudinal linear superfields. The case $s=1$ corresponds to linearized supergravity (see [10, 11] for reviews).

The gauge transformations for the superfields $H_{\alpha(s)\dot\alpha(s)}$, $\Gamma_{\alpha(s-1)\dot\alpha(s-1)}$ and $G_{\alpha(s-1)\dot\alpha(s-1)}$ postulated in [14] are

$$\delta H_{\alpha(s)\dot\alpha(s)} = g_{\alpha(s)\dot\alpha(s)} + \bar g_{\alpha(s)\dot\alpha(s)}\,, \tag{2.7}$$

$$\delta\Gamma_{\alpha(s-1)\dot\alpha(s-1)} = \frac{1}{2}\,\frac{s}{s+1}\,\bar D^{\dot\beta} D^{\beta} \bar g_{\beta\alpha(s-1)\dot\beta\dot\alpha(s-1)}\,, \tag{2.8}$$

$$\delta G_{\alpha(s-1)\dot\alpha(s-1)} = \frac{1}{2}\,\frac{s}{s+1}\,D^{\beta} \bar D^{\dot\beta} g_{\beta\alpha(s-1)\dot\beta\dot\alpha(s-1)} + {\rm i}\, s\, \partial^{\beta\dot\beta} g_{\beta\alpha(s-1)\dot\beta\dot\alpha(s-1)}\,, \tag{2.9}$$

with a longitudinal linear parameter $g_{\alpha(s)\dot\alpha(s)}$. It can be seen that $\delta G_{\alpha(s-1)\dot\alpha(s-1)}$ is longitudinal linear. Eq. (A.1) defines the action invariant under the gauge transformations (2.7) and (2.8). Similarly, eq. (A.2) defines the action invariant under the gauge transformations (2.7) and (2.9).

Two formulations of ref. [15] for the massless multiplet of an integer superspin s (with $s=1,2,\ldots$), longitudinal and transversal, contain the following dynamical variables respectively:

$$\mathcal{V}^{\perp}_{s} = \Big\{ H_{\alpha(s-1)\dot\alpha(s-1)}\,,\ \Gamma_{\alpha(s)\dot\alpha(s)}\,,\ \bar\Gamma_{\alpha(s)\dot\alpha(s)} \Big\}\,, \tag{2.10}$$

$$\mathcal{V}^{\parallel}_{s} = \Big\{ H_{\alpha(s-1)\dot\alpha(s-1)}\,,\ G_{\alpha(s)\dot\alpha(s)}\,,\ \bar G_{\alpha(s)\dot\alpha(s)} \Big\}\,. \tag{2.11}$$

Here $H_{\alpha(s-1)\dot\alpha(s-1)}$ is real, $\Gamma_{\alpha(s)\dot\alpha(s)}$ transverse linear and $G_{\alpha(s)\dot\alpha(s)}$ longitudinal linear tensor superfields. The case $s=1$ corresponds to the gravitino multiplet (see [11, 10] for reviews).

The gauge transformations for the superfields $H_{\alpha(s-1)\dot\alpha(s-1)}$, $G_{\alpha(s)\dot\alpha(s)}$ and $\Gamma_{\alpha(s)\dot\alpha(s)}$ postulated in [15] are

$$\delta H_{\alpha(s-1)\dot\alpha(s-1)} = \gamma_{\alpha(s-1)\dot\alpha(s-1)} + \bar\gamma_{\alpha(s-1)\dot\alpha(s-1)}\,, \tag{2.12}$$

$$\delta\Gamma_{\alpha(s)\dot\alpha(s)} = \frac{1}{2} D_{(\alpha_s} \bar D_{(\dot\alpha_s}\, \gamma_{\alpha_1\ldots\alpha_{s-1})\dot\alpha_1\ldots\dot\alpha_{s-1})} - {\rm i}\, s\, \partial_{(\alpha_s(\dot\alpha_s}\, \gamma_{\alpha_1\ldots\alpha_{s-1})\dot\alpha_1\ldots\dot\alpha_{s-1})}\,, \tag{2.13}$$

$$\delta G_{\alpha(s)\dot\alpha(s)} = \frac{1}{2} \bar D_{(\dot\alpha_s} D_{(\alpha_s}\, \bar\gamma_{\alpha_1\ldots\alpha_{s-1})\dot\alpha_1\ldots\dot\alpha_{s-1})}\,, \tag{2.14}$$

with a transverse linear parameter $\gamma_{\alpha(s-1)\dot\alpha(s-1)}$. It can be seen that $\delta\Gamma_{\alpha(s)\dot\alpha(s)}$ is transverse linear. Eq. (A.3) defines the action invariant under the gauge transformations (2.12) and (2.14). Similarly, eq. (A.4) defines the action invariant under the gauge transformations (2.12) and (2.13).

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3. Twistor oscillator realization

In the present section, we describe a twistor-like oscillator realization that is intrinsically related to the superfield structure of the dynamical variables and gauge transformations reviewed in the previous section. This oscillator realization can be used to obtain a generating formulation for the massless multiplets of arbitrary superspin.

Associated with the left spinor representation $(1/2, 0)$ is a pair of bosonic annihilation a^α and creation c_β operators,

$$[a_\alpha\,,\, a_\beta] = [c_\alpha\,,\, c_\beta] = 0\,, \qquad [a^\alpha\,,\, c_\beta] = \delta^\alpha{}_\beta\,, \qquad \alpha, \beta = 1, 2\,. \tag{3.1}$$

Similarly, associated with the right spinor representation $(0, 1/2)$ is a pair of bosonic annihilation $\bar{a}^{\dot\alpha}$ and creation $\bar{c}_{\dot\beta}$ operators,

$$[\bar{a}_{\dot\alpha}\,,\, \bar{a}_{\dot\beta}] = [\bar{c}_{\dot\alpha}\,,\, \bar{c}_{\dot\beta}] = 0\,, \qquad [\bar{a}^{\dot\alpha}\,,\, \bar{c}_{\dot\beta}] = \delta^{\dot\alpha}{}_{\dot\beta}\,, \qquad \dot\alpha, \dot\beta = 1, 2\,. \tag{3.2}$$

The left spinor and the right spinor operators are defined to commute with each other. The ket $|\,0\rangle$ and bra $\langle 0\,|$ vacuum states are defined by

$$a_\alpha\,|\,0\rangle = \bar{a}_{\dot\alpha}\,|\,0\rangle = 0\,, \qquad \langle 0\,|\,c_\alpha = \langle 0\,|\,\bar{c}_{\dot\alpha} = 0\,, \qquad \langle 0\,|\,0\rangle = 1\,. \tag{3.3}$$

The above commutation relations occur upon the canonical quantization of a conformally invariant twistor dynamical system, see [32] and references therein. It is quite remarkable that the same oscillator realization turns out to be dictated by the superfield structure of the gauge massless higher superspin multiplets.[4]

Along with the annihilation/creation operators introduced, let us also consider the superspace spinor covariant derivatives,

$$\{D_\alpha\,,\, D_\beta\} = \{\bar{D}_{\dot\alpha}\,,\, \bar{D}_{\dot\beta}\} = 0\,, \qquad \{D_\alpha\,,\, \bar{D}_{\dot\beta}\} = -2\mathrm{i}\,\partial_{\alpha\dot\alpha}\,. \tag{3.4}$$

We can now define the operators $\boldsymbol{\mathcal{C}} \equiv c^\alpha D_\alpha$ and $\boldsymbol{\mathcal{A}} \equiv a^\alpha D_\alpha$ with the following properties

$$\boldsymbol{\mathcal{C}}^2 = 0\,, \qquad \boldsymbol{\mathcal{A}}^2 = 0\,, \qquad \{\boldsymbol{\mathcal{C}}\,,\, \boldsymbol{\mathcal{A}}\} = D^2\,. \tag{3.5}$$

Similar properties hold for the operators $\overline{\boldsymbol{\mathcal{C}}} = \bar{c}^{\dot\alpha}\bar{D}_{\dot\alpha}$ and $\overline{\boldsymbol{\mathcal{A}}} = \bar{a}^{\dot\alpha}\bar{D}_{\dot\alpha}$,

$$\overline{\boldsymbol{\mathcal{C}}}^2 = 0\,, \qquad \overline{\boldsymbol{\mathcal{A}}}^2 = 0\,, \qquad \{\overline{\boldsymbol{\mathcal{C}}}\,,\, \overline{\boldsymbol{\mathcal{A}}}\} = -\bar{D}^2\,. \tag{3.6}$$

Consider a state $|\Psi_n\rangle$ in the Fock space of the form

$$|\Psi_n\rangle = \Psi_{\alpha_1\cdots\alpha_n}(z)\, c^{\alpha_1}\cdots c^{\alpha_n}|\,0\rangle\,, \qquad \Psi_{(\alpha_1\cdots\alpha_n)} = \Psi_{\alpha_1\cdots\alpha_n}\,. \tag{3.7}$$

Since

$$\begin{aligned} \boldsymbol{\mathcal{C}}\,|\Psi_n\rangle &= D_{(\alpha_1}\Psi_{\alpha_2\cdots\alpha_{n+1})}\, c^{\alpha_1}\cdots c^{a_{n+1}}|\,0\rangle\,, \\ \boldsymbol{\mathcal{A}}|\Psi_n\rangle &= n\,D^\beta\Psi_{\beta\alpha_1\cdots\alpha_{n-1}}\, c^{\alpha_1}\cdots c^{a_{n-1}}|\,0\rangle\,, \end{aligned} \tag{3.8}$$

[4]In Vasiliev's approach to nonlinear higher spin equations of motion, one often considers a smaller set of oscillators: $[y_\alpha\,,\, y_\beta] = 2\mathrm{i}\,\varepsilon_{\alpha\beta}$, $[\bar{y}_{\dot\alpha}\,,\, \bar{y}_{\dot\beta}] = 2\mathrm{i}\,\varepsilon_{\dot\alpha\dot\beta}$, $[y_\alpha\,,\, \bar{y}_{\dot\beta}] = 0$, see e.g. [33].

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we obtain

$$
\begin{aligned}
\mathcal{C}\,|\Psi_n\rangle &= 0 \quad \longleftrightarrow \quad D_{(\alpha_1}\Psi_{\alpha_2\cdots\alpha_{n+1})} = 0\,,\\
\mathcal{A}|\Psi_n\rangle &= 0 \quad \longleftrightarrow \quad D^{\beta}\Psi_{\beta\alpha_1\cdots\alpha_{n-1}} = 0\,.
\end{aligned}
\tag{3.9}
$$

Now, it is obvious that the transverse and longitudinal linear superfields, which were introduced in the previous section, are intrinsically related to the operators $\overline{\mathcal{C}}$ and $\overline{\mathcal{A}}$. Consider states in the Fock space of the form

$$
\begin{aligned}
|\Psi_{(k,l)}\rangle &= \Psi_{(\alpha_1\cdots\alpha_k)(\dot\alpha_1\cdots\dot\alpha_l)}\, c^{\alpha_1}\cdots c^{\alpha_k}\,\bar c^{\dot\alpha_1}\cdots\bar c^{\dot\alpha_l}\,|\,0\rangle\,,\\
\langle\Psi_{(k,l)}| &= \langle 0\,|\,a^{\alpha_1}\cdots a^{\alpha_k}\,\bar a^{\dot\alpha_1}\cdots\bar a^{\dot\alpha_l}\,\Psi_{(\alpha_1\cdots\alpha_k)(\dot\alpha_1\cdots\dot\alpha_l)}\,.
\end{aligned}
\tag{3.10}
$$

Then, the constraint (2.1) is equivalent to

$$
\overline{\mathcal{A}}|\Gamma_{(k,l)}\rangle = 0 \quad \longleftrightarrow \quad \langle\Gamma_{(k,l)}|\,\overline{\mathcal{C}} = 0\,. \tag{3.11}
$$

Similarly, the constraint (2.2) is equivalent to

$$
\overline{\mathcal{C}}\,|G_{(k,l)}\rangle = 0 \quad \longleftrightarrow \quad \langle G_{(k,l)}|\,\overline{\mathcal{A}} = 0\,. \tag{3.12}
$$

Relations (3.5) are reminiscent of famous constructions in differential geometry, see e.g. [34]. One can consider $\mathcal{C}$ and $\mathcal{A}$ to be analogues of the exterior differential d and the co-differential $\delta \propto *\,d\,*$, with $*$ the Hodge star operation. Then, the third relation (3.5) is analogous to the definition of the laplacian $\{d\,,\delta\} = \Delta$. Of course, for this analogy to be quite solid, it would be good to have a 'star' operation $*$ in superspace[5] with the properties

$$
\mathcal{A} \;\propto\; *\,\mathcal{C}\,*\,, \qquad **=\mathrm{id}\,. \tag{3.13}
$$

Such an operation does exist, and it exchanges the ket and bra states.

$$
*:\; |\Psi_{(k,l)}\rangle \quad \longrightarrow \quad \langle\Psi_{(k,l)}|\,. \tag{3.14}
$$

The specific features of the superspace construction, as compared with that of differential geometry, are given by the identities

$$
\mathcal{C}\,\mathcal{A} = -\frac{1}{2}\,\mathcal{N}\,D^2\,, \qquad \overline{\mathcal{C}}\,\overline{\mathcal{A}} = \frac{1}{2}\,\overline{\mathcal{N}}\,\bar D^2\,, \tag{3.15}
$$

with $\mathcal{N}$ and $\overline{\mathcal{N}}$ the number operators

$$
\begin{aligned}
\mathcal{N} &= c_\alpha\,a^\alpha\,, \qquad \mathcal{N}\,|\Psi_{(k,l)}\rangle = k\,|\Psi_{(k,l)}\rangle\,;\\
\overline{\mathcal{N}} &= \bar c_{\dot\alpha}\,\bar a^{\dot\alpha}\,, \qquad \overline{\mathcal{N}}\,|\Psi_{(k,l)}\rangle = l\,|\Psi_{(k,l)}\rangle\,.
\end{aligned}
\tag{3.16}
$$

Now, taking into account the obvious identities

$$
\begin{aligned}
\{\mathcal{C}\,,\overline{\mathcal{C}}\} &= -2\mathrm{i}\,c^\alpha\,\bar c^{\dot\alpha}\,\partial_{\alpha\dot\alpha}\,, \qquad \{\mathcal{C}\,,\overline{\mathcal{A}}\} = -2\mathrm{i}\,c^\alpha\,\bar a^{\dot\alpha}\,\partial_{\alpha\dot\alpha}\,,\\
\{\mathcal{A},\overline{\mathcal{C}}\} &= -2\mathrm{i}\,a^\alpha\,\bar c^{\dot\alpha}\,\partial_{\alpha\dot\alpha}\,, \qquad \{\mathcal{A},\overline{\mathcal{A}}\} = -2\mathrm{i}\,a^\alpha\,\bar a^{\dot\alpha}\,\partial_{\alpha\dot\alpha}\,,
\end{aligned}
\tag{3.17}
$$

[5] In the work of ref. [35] it was noted there is a natural definition for the Hodge 'star' operation defined on the irreducible pre-potentials of lower spin 4D, $\mathcal{N}=1$ gauge theories.

one can rewrite the higher superspin gauge transformations in terms of Fock space states and the differential operators $\boldsymbol{\mathcal{C}}$, $\boldsymbol{\mathcal{A}}$, $\overline{\boldsymbol{\mathcal{C}}}$ and $\overline{\boldsymbol{\mathcal{A}}}$. In particular, the gauge transformations (2.7)–(2.9) take the form

$$\begin{aligned}
\delta|H_{(s,s)}\rangle &= \overline{\boldsymbol{\mathcal{C}}}\,|\zeta_{(s,s-1)}\rangle - \boldsymbol{\mathcal{C}}\,|\bar{\zeta}_{(s-1,s)}\rangle\,, \\
\delta|\Gamma_{(s-1,s-1)}\rangle &= -\frac{1}{2}\,\frac{1}{s(s+1)}\,\overline{\boldsymbol{\mathcal{A}}}\boldsymbol{\mathcal{A}}\boldsymbol{\mathcal{C}}\,|\bar{\zeta}_{(s-1,s)}\rangle\,, \\
\delta|G_{(s-1,s-1)}\rangle &= -\frac{1}{s^2(s+1)}\Big(\boldsymbol{\mathcal{A}}\overline{\boldsymbol{\mathcal{A}}} + (s+1)\,\overline{\boldsymbol{\mathcal{A}}}\boldsymbol{\mathcal{A}}\Big)\overline{\boldsymbol{\mathcal{C}}}\,|\zeta_{(s,s-1)}\rangle\,,
\end{aligned} \tag{3.18}$$

with $\zeta_{\alpha(s)\dot{\alpha}(s-1)}$ an unconstrained spin-tensor. Similar results follow for the gauge transformations (2.12)–(2.14). One can express the gauge-invariant actions (A.1)–(A.4) in terms of the operators $\boldsymbol{\mathcal{C}}$, $\boldsymbol{\mathcal{A}}$, $\overline{\boldsymbol{\mathcal{C}}}$ and $\overline{\boldsymbol{\mathcal{A}}}$, and special Fock space states. In particular, one obtains

$$\begin{aligned}
D^\alpha \bar{D}^2 D_\alpha|\Psi_{(k,l)}\rangle &= \frac{1}{k+1}\Big(\boldsymbol{\mathcal{C}}\,\{\overline{\boldsymbol{\mathcal{C}}}\,,\overline{\boldsymbol{\mathcal{A}}}\}\,\boldsymbol{\mathcal{A}} - \boldsymbol{\mathcal{A}}\,\{\overline{\boldsymbol{\mathcal{C}}}\,,\overline{\boldsymbol{\mathcal{A}}}\}\,\boldsymbol{\mathcal{C}}\Big)\,|\Psi_{(k,l)}\rangle \\
&= \frac{1}{l+1}\Big(\overline{\boldsymbol{\mathcal{C}}}\,\{\boldsymbol{\mathcal{C}}\,,\boldsymbol{\mathcal{A}}\}\,\overline{\boldsymbol{\mathcal{A}}} - \overline{\boldsymbol{\mathcal{A}}}\,\{\boldsymbol{\mathcal{C}}\,,\boldsymbol{\mathcal{A}}\}\,\overline{\boldsymbol{\mathcal{C}}}\Big)\,|\Psi_{(k,l)}\rangle\,.
\end{aligned} \tag{3.19}$$

Thus, it is clear that the Fock space realization discussed in this section can be used as an organizing tool for formulating the dynamics of the higher spin gauge superspins. Moreover, this twistor formalism can be used to obtain a generating formulation (different from the formulation developed in [20]) for such supermultiplets.

Of course, the twistor construction discussed above also appears (albeit in a hidden manner) in the context of more familiar 4D, $\mathcal{N}=1$ theories. Consider, for instance, two Fock space states $|V\rangle \equiv |V_{(0,0)}\rangle$ and $|\Lambda\rangle \equiv |\Lambda_{(0,0)}\rangle$, with V real and Λ chiral, $\overline{\boldsymbol{\mathcal{C}}}|\Lambda\rangle = 0$. It is now clear that 4D, $\mathcal{N}=1$ abelian gauge theory, in this language, takes the form (i) $\delta\,|V\rangle = |\Lambda\rangle + |\bar{\Lambda}\rangle$ for the gauge transformation; (ii) $|W_{(1,0)}\rangle = -\frac{1}{4}\overline{\boldsymbol{\mathcal{A}}}\,\overline{\boldsymbol{\mathcal{C}}}\,\boldsymbol{\mathcal{C}}|V\rangle$ for the usual field strength; and (iii) the result in (3.19) in the case of $k=l=0$ for the equation of motion. It is thus clear that the quartet of BRST-like operators $\boldsymbol{\mathcal{C}}$, $\boldsymbol{\mathcal{A}}$, $\overline{\boldsymbol{\mathcal{C}}}$ and $\overline{\boldsymbol{\mathcal{A}}}$ defined in terms of the twistor annihilation and creation operators can be used to express usual 4D, $\mathcal{N}=1$ supersymmetric gauge theories as statements on an associated Fock space.

It is worth pointing out that the operators (3.17) have interesting interpretations when the states in (3.10) are associated with (gauge) differential forms (say, a zero-form $|\varphi_{(0,0)}\rangle$, a one-form $|V_{(1,1)}\rangle$, two-forms $|F_{(2,0)}\rangle$ and $|\bar{F}_{(0,2)}\rangle$) for which the operators d and δ are defined. For such states

(a.) the operator $\{\boldsymbol{\mathcal{A}}\,,\overline{\boldsymbol{\mathcal{A}}}\}$ generates the effect of δ;

(b.) the operators $\{\boldsymbol{\mathcal{C}}\,,\overline{\boldsymbol{\mathcal{A}}}\}$ and $\{\boldsymbol{\mathcal{A}}\,,\overline{\boldsymbol{\mathcal{C}}}\}$ generate the (building blocks for) gauge invariant field strengths and Bianchi identities; and

(c.) the operator $\{\boldsymbol{\mathcal{C}}\,,\overline{\boldsymbol{\mathcal{C}}}\}$ generates the the gauge transformation of the one-form.

So the quartet $\boldsymbol{\mathcal{C}}$, $\boldsymbol{\mathcal{A}}$, $\overline{\boldsymbol{\mathcal{C}}}$ and $\overline{\boldsymbol{\mathcal{A}}}$ allows for a factorization of the usual the exterior differential and the co-differential as realized in the Fock space.

As is well-known, the superfield constraints in extended super Yang-Mills theories [36] naturally lead to elegant twistorial interpretations as integrability conditions in spaces with auxiliary dimensions [37 − 39], and these and related ideas apparently culminated in the discovery of the profound concept of harmonic superspace [40]. Our discussion above demonstrates that the superfield structure of the higher spin gauge supermultiplets becomes transparent within the twistor approach. We believe that this is not accidental, and may be of importance in the context of superstring theory.

4. Field strengths and Bianchi identities: half-integer superspin

We now turn to discussing the structure of gauge-invariant field strengths and corresponding Bianchi identities, which occur in the massless higher superspin models. We first consider the case of half-integer superspin.

In both the transverse and longitudinal formulations, there exists a gauge-invariant field chiral strength that is constructed in terms of the prepotential $H_{\alpha(s)\dot\alpha(s)}$ only. It has the form [10]

$$W_{\alpha(2s+1)} = \frac{1}{4}\bar D^2 \partial_{(\alpha_1}{}^{\dot\beta_1} \cdots \partial_{\alpha_s}{}^{\dot\beta_s} D_{\alpha_{2s+1}} H_{\alpha_{s+1}\cdots\alpha_{2s})\dot\beta_1\cdots\dot\beta_s}\,, \qquad \bar D_{\dot\beta} W_{\alpha(2s+1)} = 0\,. \tag{4.1}$$

The other functional-independent strengths involve, depending upon the formulation under consideration, the compensators $\Gamma_{\alpha(s-1)\dot\alpha(s-1)}$ or $G_{\alpha(s-1)\dot\alpha(s-1)}$ and/or their conjugates. On the mass shell, $W_{\alpha(2s+1)}$ and its conjugate are the only non-vanishing gauge-invariant strengths.

4.1 Transverse formulation

The equations of motion, $E^{\perp}{}_{\alpha(s)\dot\alpha(s)} = 0$ and $L_{\alpha(s-1)\dot\alpha(s)} = 0$, are given in terms of the following gauge invariant field strengths

$$\begin{aligned}
E^{\perp}{}_{\alpha(s)\dot\alpha(s)} &= \frac{1}{4} D^\beta \bar D^2 D_\beta H_{\alpha(s)\dot\alpha(s)} + D_{\alpha_s}\bar D_{\dot\alpha_s}\Gamma_{\alpha(s-1)\dot\alpha(s-1)} - \bar D_{\dot\alpha_s} D_{\alpha_s}\bar\Gamma_{\alpha(s-1)\dot\alpha(s-1)}\,,\\
L_{\alpha(s-1)\dot\alpha(s)} &= -\frac{1}{4}\bar D^2 D^\beta H_{\beta\alpha(s-1)\dot\alpha(s)} + \bar D_{\dot\alpha_s}\left(\bar\Gamma_{\alpha(s-1)\dot\alpha(s-1)} + \frac{s+1}{s}\Gamma_{\alpha(s-1)\dot\alpha(s-1)}\right).
\end{aligned} \tag{4.2}$$

They can be shown to obey the Bianchi identity

$$D^\beta E^{\perp}{}_{\beta\alpha(s-1)\dot\alpha(s)} = \frac{1}{2} D^2 L_{\alpha(s-1)\dot\alpha(s)}\,. \tag{4.3}$$

One can also check that the fields strengths (4.1) and (4.2) are related to each other by

$$\begin{aligned}
D^\beta W_{\beta\alpha(2s)} = \partial_{(\alpha_1}{}^{\dot\beta_1}\cdots\partial_{\alpha_s}{}^{\dot\beta_s}\Big\{ & E^{\perp}{}_{\alpha_{s+1}\cdots\alpha_{2s})\dot\beta(s)} + \\
& + \frac{s}{2s+1}\Big(\bar D_{\dot\beta_s}\bar L_{\alpha_{s+1}\cdots\alpha_{2s})\dot\beta(s-1)} - \\
& - D_{\alpha_{s+1}} L_{\alpha_{s+2}\cdots\alpha_{2s})\dot\beta(s)}\Big)\Big\}\,.
\end{aligned} \tag{4.4}$$

On-shell, this turns into $D^\beta W_{\beta\alpha(2s)} = 0$. As discussed in detail in [11], the equations $\bar D_{\dot\beta} W_{\alpha(2s+1)} = D^\beta W_{\beta\alpha(2s)} = 0$ define an irreducible on-shell massless superfield.

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4.2 Longitudinal formulation

The equations of motion, ${E^{\|}}_{\alpha(s)\dot\alpha(s)} = 0$ and $T_{\alpha(s-2)\dot\alpha(s-1)} = 0$, are given in terms of the following gauge invariant field strengths

$$
\begin{aligned}
{E^{\|}}_{\alpha(s)\dot\alpha(s)} &= \frac{1}{4} D^\beta \bar D^2 D_\beta H_{\alpha(s)\dot\alpha(s)} - \frac{1}{4}\,\frac{s}{2s+1}[D_{\alpha_s}, \bar D_{\dot\alpha_s}]\,[D^\beta, \bar D^{\dot\beta}] H_{\alpha(s-1)\beta\dot\alpha(s-1)\dot\beta} - \\
&\quad - s\,\partial_{a_s\dot\alpha_s}\,\partial^{\beta\dot\beta} H_{\alpha(s-1)\beta\dot\alpha(s-1)\dot\beta} - \\
&\quad - 2\mathrm{i}\,\frac{s}{2s+1}\,\partial_{\alpha_s\dot\alpha_s}\left(G_{\alpha(s-1)\dot\alpha(s-1)} - \bar G_{\alpha(s-1)\dot\alpha(s-1)}\right), \qquad (4.5)\\
T_{\alpha(s-1)\dot\alpha(s-2)} &= \bar D^{\dot\beta}\left(\mathrm{i}s\,\partial^{\gamma\dot\gamma} H_{\gamma\alpha(s-1)\,\dot\beta\dot\gamma\dot\alpha(s-2)} + \bar G_{\alpha(s-1)\dot\beta\dot\alpha(s-2)} - \frac{s+1}{s} G_{\alpha(s-1)\dot\beta\dot\alpha(s-2)}\right).
\end{aligned}
$$

The field strengths introduced obey the Bianchi identity

$$
D^\beta {E^{\|}}_{\beta\alpha(s-1)\dot\alpha(s)} = \frac{1}{2s+1}\left\{\bar D_{\dot\alpha_s} D_{\alpha_{s-1}} \bar T_{\alpha(s-2)\dot\alpha(s-1)} - 2\mathrm{i}(s-1)\partial_{\alpha_{s-1}\dot\alpha_s}\bar T_{\alpha(s-2)\dot\alpha(s-1)}\right\}. \qquad (4.6)
$$

The Bianchi identity relating the strengths (4.1) and (4.5) is

$$
D^\beta W_{\beta\alpha(2s)} = \partial_{(\alpha_1}{}^{\dot\beta_1}\cdots\partial_{\alpha_s}{}^{\dot\beta_s}\,{E^{\|}}_{\alpha_{s+1}\cdots\alpha_{2s})\dot\beta(s)}\,. \qquad (4.7)
$$

5. Field strengths and Bianchi identities: integer superspin

Here we consider only the longitudinal formulation. Let us introduce the completely symmetric tensor [15]

$$
\begin{aligned}
W_{\alpha(2s)} &= \frac{1}{2} s\,\partial_{(\alpha_1}{}^{\dot\beta_1}\cdots\partial_{\alpha_{s-1}}{}^{\dot\beta_{s-1}}\bar D^{\dot\beta_s} D_{\alpha_s} G_{\alpha_{s+1}\cdots\alpha_{2s})\dot\beta_1\cdots\dot\beta_s} - \\
&\quad - \mathrm{i}\,\partial_{(\alpha_1}{}^{\dot\beta_1}\cdots\partial_{\alpha_s}{}^{\dot\beta_s} G_{\alpha_{s+1}\cdots\alpha_{2s})\dot\beta_1\cdots\dot\beta_s}\,. \qquad (5.1)
\end{aligned}
$$

It can be readily checked that $W_{\alpha(2s)}$ is gauge invariant, and that it is chiral,

$$
\bar D_{\dot\beta} W_{\alpha(2s)} = 0\,. \qquad (5.2)
$$

If one introduces an unconstrained gauge prepotentil for $G_{\alpha(s)\dot\alpha(s)}$,

$$
G_{\alpha(s)\dot\alpha(s)} = \bar D_{(\dot\alpha_s}\Psi_{\alpha(s)\dot\alpha_1\cdots\dot\alpha_{s-1})}\,, \qquad (5.3)
$$

then the field strength can be expressed in the form [10]

$$
W_{\alpha(2s)} = \frac{1}{4}(s+1)\bar D^2 \partial_{(\alpha_1}{}^{\dot\beta_1}\cdots\partial_{\alpha_{s-1}}{}^{\dot\beta_{s-1}} D_{\alpha_s}\Psi_{\alpha_{s+1}\cdots\alpha_{2s})\dot\beta_1\cdots\dot\beta_{s-1}}\,. \qquad (5.4)
$$

The equations of motion, ${E^{\|}}_{\alpha(s-1)\dot\alpha(s-1)} = 0$ and $T_{\alpha(s)\dot\alpha(s-1)} = 0$, are given in terms of the following gauge invariant field strengths

$$
\begin{aligned}
{E^{\|}}_{\alpha(s-1)\dot\alpha(s-1)} &= \frac{1}{4} D^\beta \bar D^2 D_\beta H_{\alpha(s-1)\dot\alpha(s-1)} + \\
&\quad + \frac{s}{s+1}\left(D^\beta \bar D^{\dot\beta} G_{\beta\alpha(s-1)\dot\beta\dot\alpha(s-1)} - \bar D^{\dot\beta} D^\beta \bar G_{\beta\alpha(s-1)\dot\beta\dot\alpha(s-1)}\right),\\
T_{\alpha(s)\dot\alpha(s-1)} &= \bar D^{\dot\beta}\Big(-\frac{1}{2}\,\frac{s}{s+1}\,\bar D_{(\dot\beta} D_{(\alpha_s} H_{\alpha_1\cdots\alpha_{s-1})\,\dot\alpha_1\cdots\dot\alpha_{s-1})} + \\
&\quad + \bar G_{\alpha(s)\dot\beta\dot\alpha(s-1)} + \frac{s}{s+1} G_{\alpha(s)\dot\beta\dot\alpha(s-1)}\Big). \qquad (5.5)
\end{aligned}
$$

The Bianchi identities are:

$$
\begin{aligned}
D_{(\alpha_s} E^{\parallel}{}_{\alpha_1\cdots\alpha_{s-1})\dot\beta(s-1)} &= -\frac{1}{2} D^2 T_{\alpha(s)\dot\alpha(s-1)}\,, \\
D^\gamma W_{\gamma\alpha(2s-1)} &= -\frac{1}{8}(s+1)\Big(\partial_{(\alpha_1}{}^{\dot\beta_1}\cdots\partial_{\alpha_{s-1}}{}^{\dot\beta_{s-1}} D^2 T_{\alpha_s\cdots\alpha_{2s-1})\dot\beta(s-1)} - \\
&\qquad -2\partial_{(\alpha_1}{}^{\dot\beta_1}\cdots\partial_{\alpha_{s-1}}{}^{\dot\beta_{s-1}}\{\bar D^{\dot\beta_s} D_{\alpha_s} - 2\mathrm{i}\partial_{\alpha_s}{}^{\dot\beta_s}\}\bar T_{\alpha_{s+1}\cdots\alpha_{2s-1})\dot\beta(s)}\Big).
\end{aligned} \tag{5.6}
$$

On-shell, the latter turns into $D^\beta W_{\beta\alpha(2s-1)} = 0$, and therefore $W_{\alpha(2s)}$ becomes an irreducible on-shell massless superfield [11].

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A. Massless higher superspin actions

In this appendix, we collect the gauge-invariant action for the massless higher superspin multiplets [14, 15].

A.1 Half-integer superspin

In the transverse formulation, the action reads

$$
\begin{aligned}
S^{\perp}_{s+1/2} = \Big(-\frac{1}{2}\Big)^s \int \mathrm{d}^8 z \Big\{ &\frac{1}{8} H^{\alpha(s)\dot\alpha(s)} D^\beta \bar D^2 D_\beta H_{\alpha(s)\dot\alpha(s)} + \\
&+ H^{\alpha(s)\dot\alpha(s)}\left(D_{\alpha_s}\bar D_{\dot\alpha_s}\Gamma_{\alpha(s-1)\dot\alpha(s-1)} - \bar D_{\dot\alpha_s} D_{\alpha_s}\bar\Gamma_{\alpha(s-1)\dot\alpha(s-1)}\right) + \\
&+ \Big(\bar\Gamma\cdot\Gamma + \frac{s+1}{s}\Gamma\cdot\Gamma + \text{c.c.}\Big)\Big\}.
\end{aligned} \tag{A.1}
$$

In the longitudinal formulation, the action is

$$
\begin{aligned}
S^{\parallel}_{s+1/2} = \Big(-\frac{1}{2}\Big)^s \int \mathrm{d}^8 z \Big\{ &\frac{1}{8} H^{\alpha(s)\dot\alpha(s)} D^\beta \bar D^2 D_\beta H_{\alpha(s)\dot\alpha(s)} - \\
&- \frac{1}{8}\frac{s}{2s+1}\Big([D_\gamma,\bar D_{\dot\gamma}] H^{\gamma\alpha(s-1)\dot\gamma\dot\alpha(s-1)}\Big)[D^\beta,\bar D^{\dot\beta}] H_{\beta\alpha(s-1)\dot\beta\dot\alpha(s-1)} + \\
&+ \frac{s}{2}\Big(\partial_{\dot\gamma} H^{\gamma\alpha(s-1)\dot\gamma\dot\alpha(s-1)}\Big)\partial^{\beta\dot\beta} H_{\beta\alpha(s-1)\dot\beta\dot\alpha(s-1)} \\
&+ 2\mathrm{i}\frac{s}{2s+1}\partial_{\gamma\dot\gamma} H^{\gamma\alpha(s-1)\dot\gamma\dot\alpha(s-1)}\Big(G_{\alpha(s-1)\dot\alpha(s-1)} - \bar G_{\alpha(s-1)\dot\alpha(s-1)}\Big) + \\
&+ \frac{1}{2s+1}\Big(\bar G\cdot G - \frac{s+1}{s} G\cdot G + \text{c.c.}\Big)\Big\}.
\end{aligned} \tag{A.2}
$$

The models (A.1) and (A.2) are dually equivalent [14].

A.2 Integer superspin

In the longitudinal formulation, the action is

$$S_s^{\parallel} = \Big(-\frac{1}{2}\Big)^s \int \mathrm{d}^8 z \Big\{\frac{1}{8} H^{\alpha(s-1)\dot\alpha(s-1)} D^\beta \bar D^2 D_\beta H_{\alpha(s-1)\dot\alpha(s-1)} + \\ + \frac{s}{s+1} H^{\alpha(s-1)\dot\alpha(s-1)} \Big(D^\beta \bar D^{\dot\beta} G_{\beta\alpha(s-1)\dot\beta\dot\alpha(s-1)} - \\ - \bar D^{\dot\beta} D^\beta \bar G_{\beta\alpha(s-1)\dot\beta\dot\alpha(s-1)}\Big) + \\ + \Big(\bar G \cdot G + \frac{s}{s+1} G\cdot G + \mathrm{c.c.}\Big)\Big\}, \tag{A.3}$$

while the action in the transversal formulation takes the form

$$S_s^{\perp} = -\Big(-\frac{1}{2}\Big)^s \int \mathrm{d}^8 z \Big\{-\frac{1}{8} H^{\alpha(s-1)\dot\alpha(s-1)} D^\beta \bar D^2 D_\beta H_{\alpha(s-1)\dot\alpha(s-1)} + \\ + \frac{1}{8}\frac{s^2}{(s+1)(2s+1)} \Big([D^{\alpha_s}, \bar D^{\dot\alpha_s}] H^{\alpha(s-1)\dot\alpha(s-1)}\Big) \times \\ \times [D_{(\alpha_s}, \bar D_{(\dot\alpha_s}] H_{\alpha_1\dots\alpha_{s-1})\dot\alpha_1\dots\dot\alpha_{s-1})} + \\ + \frac{1}{2}\frac{s^2}{s+1}\Big(\partial^{\alpha_s\dot\alpha_s} H^{\alpha(s-1)\dot\alpha(s-1)}\Big)\partial_{(\alpha_s(\dot\alpha_s} H_{\alpha_1\dots\alpha_{s-1})\dot\alpha_1\dots\dot\alpha_{s-1})} + \\ + 2\mathrm{i}\frac{s}{2s+1} H^{\alpha(s-1)\dot\alpha(s-1)} \partial^{\alpha_s\dot\alpha_s}\Big(\Gamma_{\alpha(s)\dot\alpha(s)} - \bar\Gamma_{\alpha(s)\dot\alpha(s)}\Big) + \\ + \frac{1}{2s+1}\Big(\bar\Gamma\cdot\Gamma - \frac{s+1}{s}\Gamma\cdot\Gamma + \mathrm{c.c.}\Big)\Big\}. \tag{A.4}$$

The models (A.3) and (A.4) are dually equivalent [15].

B. Dual formulations for the massive vector multiplet

This appendix contains three dually equivalent realizations for the massive gravitino multiplet. Only the first realization possesses the property that the compensating multiplet can be completely gauged away.

Consider the Stückelberg formulation for the massive vector multiplet

$$S_{\rm I} = \frac{1}{4}\int \mathrm{d}^6 z\, W^\alpha W_\alpha + \frac{1}{2} m^2 \int \mathrm{d}^8 z\, V^2 + \int \mathrm{d}^8 z\, \bar\Phi \Phi - m \int \mathrm{d}^8 z\, V\, (\bar\Phi + \Phi)\,, \tag{B.1}$$

with $W_\alpha = -\frac{1}{4}\bar D^2 D_\alpha V$, and Φ a chiral superfield, $\bar D_{\dot\alpha}\Phi = 0$. The action is invariant under the gauge transformations

$$\delta V = \Lambda + \bar\Lambda\,, \qquad \delta\Phi = m\,\Lambda\,, \qquad \bar D_{\dot\alpha}\Lambda = 0\,. \tag{B.2}$$

This gauge freedom can be used to gauge away Φ.

The model (B.1) possesses a dual formulation in which the chiral multiplet is replaced by the tensor multiplet described by a real linear superfield

$$L = \frac{1}{2}(D^\alpha \eta_\alpha + \bar D_{\dot\alpha}\bar\eta^{\dot\alpha})\,, \qquad \bar D_{\dot\alpha}\eta_\alpha = 0\,, \tag{B.3}$$

with η_α an unconstrained chiral spinor superfield. The dual action[6] is [42]

$$
\begin{aligned}
S_{\rm II} &= \frac{1}{4}\int \mathrm{d}^6z\, W^\alpha W_\alpha - \frac{1}{2}\int \mathrm{d}^8z L^2 + m\int \mathrm{d}^8z L\, V \\
&= \frac{1}{4}\int \mathrm{d}^6z\, W^\alpha W_\alpha - \frac{1}{2}\int \mathrm{d}^8z L^2 - \frac{1}{2}m\left\{\int \mathrm{d}^6z\, W^\alpha\eta_\alpha + \text{c.c.}\right\}.
\end{aligned}
\tag{B.4}
$$

This action remains invariant under the following gauge transformations

$$
\begin{aligned}
\delta V &= \Lambda + \bar\Lambda\,, \qquad \bar D_{\dot\alpha}\Lambda = 0 \\
\delta\eta_\alpha &= \mathrm{i}\,\bar D^2 D_\alpha K\,, \qquad \bar K = K
\end{aligned}
\tag{B.5}
$$

which are characteristic of the massless vector multiplet and the massless tensor multiplet respectively. Unlike the realization (B.1), the chiral spinor compensator η_α cannot be gauged away even on the mass shell.

The model (B.1) possesses another dual formulation in which the chiral multiplet is replaced by the so-called nonminimal scalar multiplet described by a complex linear superfield

$$
\Gamma = \bar D_{\dot\alpha}\bar\Upsilon^{\dot\alpha}\,, \tag{B.6}
$$

with Υ_α an unconstrained spinor superfield.

$$
S_{\rm III} = \frac{1}{4}\int \mathrm{d}^6z\, W^\alpha W_\alpha - \frac{1}{2}m^2\int \mathrm{d}^8z\, V^2 - \int \mathrm{d}^8z\,\bar\Gamma\Gamma + m\int \mathrm{d}^8z\, V\,(\bar\Gamma + \Gamma)\,. \tag{B.7}
$$

The corresponding gauge invariance is

$$
\delta V = \Lambda + \bar\Lambda\,, \qquad \delta\Gamma = m\,\Lambda\,, \qquad \bar D_{\dot\alpha}\Lambda = 0\,. \tag{B.8}
$$

The compensator Γ cannot be gauged away even on the mass shell.

The fate of the mass term $m^2\int \mathrm{d}^8z\, V^2$ in the models (B.1), (B.4) and (B.7) is clearly very distinctive.

C. Massive gravitino multiplet as a gauge theory

As discussed in the introduction, there are reasons to expect that a massive superspin-s multiplet can be described as a gauge-invariant dynamical system involving massless multiplets of superspins $s, s-1/2, \ldots, 0$. To our knowledge, no prior realization of this approach has been presented in 4D, $\mathcal{N}=1$ superspace for a massive superspin-s multiplet for $s > 3/2$.

In this appendix, we present the simplest nontrivial case — massive gravitino mutiplet corresponding to $s = 1$. These results will show that the program described in in the introduction works in this specific case, and thus it is encouraging for the future pursuit of such realizations of this approach for all higher superspin cases.

[6]See [43] for $\mathcal{N}=2$ supersymmetric generalizations of the models considered here.

In the case $s = 1$, the integer-superspin longitudinal formulation[7] (A.3) describes the off-shell massless gravitino multiplet introduced first in [44, 45] at the component level and then formulated in [46] in terms of superfields. The dynamical variables are (i) a real scalar superfield H and (ii) an unconstrained spinor superfield Ψ_α that generates $G_{\alpha\dot\alpha}$ according to (5.3). The action can be represented in the form

$$S^{\parallel}_{(1,3/2)}[\Psi, H] = \hat{S}[\Psi] - \frac{1}{4}\int \mathrm{d}^8z \Big\{ \Psi^\alpha \bar{D}^2 D_\alpha H + \bar{\Psi}_{\dot\alpha} D^2 \bar{D}^{\dot\alpha} H \Big\} - \frac{1}{16}\int \mathrm{d}^8z\, H D^\alpha \bar{D}^2 D_\alpha\, H\,, \tag{C.1}$$

where

$$\hat{S}[\Psi] = \int \mathrm{d}^8z \Big\{ D^\alpha \bar{\Psi}^{\dot\alpha} \bar{D}_{\dot\alpha} \Psi_\alpha - \frac{1}{4}\bar{D}^{\dot\alpha}\Psi^\alpha \bar{D}_{\dot\alpha}\Psi_\alpha - \frac{1}{4} D_\alpha \bar{\Psi}_{\dot\alpha} D^\alpha \bar{\Psi}^{\dot\alpha} \Big\}. \tag{C.2}$$

In accordance with (2.13), the gauge freedom is:

$$\delta H = D^\beta L_\beta + \bar{D}_{\dot\beta} \bar{L}^{\dot\beta}\,, \quad \delta \Psi_\alpha = \eta_\alpha + \frac{1}{2} D_\alpha D^\beta L_\beta\,, \qquad \bar{D}_{\dot\alpha}\eta_\alpha = 0\,, \tag{C.3}$$

with the gauge parameter L_α being an unconstrained spinor. It is obvious that H can be completely gauged away.

The massive extension of (C.1) was obtained in [29, 28]

$$S[\Psi, H] = S^{\parallel}_{(1,3/2)}[\Psi, H] + m \int \mathrm{d}^8z \Big\{ \Psi^2 + \bar{\Psi}^2 - \frac{1}{4} m\, H^2 + \frac{1}{2} H \Big(D\Psi + \bar{D}\bar{\Psi} \Big) \Big\}. \tag{C.4}$$

In this model for massive gravitino multilet, it turns out that the degrees of freedom associated with H can be integrated out. Indeed, let us implement the following transformation

$$\Psi_\alpha \to \tilde{\Psi}_\alpha = \Psi_\alpha + \frac{1}{16m}\, \bar{D}^2 D_\alpha\, H \tag{C.5}$$

in the action. This leads to

$$\begin{aligned} S[\tilde{\Psi}, H] \,=\, & \hat{S}[\Psi] - \frac{1}{4} m^2 \int \mathrm{d}^8z \Big\{ H^2 - \frac{2}{m} H \Big(D^\alpha \Big(\Psi_\alpha + \frac{1}{4m}\, \bar{D}^2\, \Psi_\alpha \Big) + \text{c.c.} \Big) \Big\} + \\ & + m \int \mathrm{d}^8z \Big\{ \Psi^2 + \bar{\Psi}^2 \Big\}. \end{aligned} \tag{C.6}$$

As is seen, the superfield H has become auxiliary, and therefore it can be eliminated. In conjunction with the shift

$$\Psi_\alpha \to \Psi_\alpha - \frac{1}{4m}\, \bar{D}^2\, \Psi_\alpha\,, \tag{C.7}$$

one then ends up with

$$\begin{aligned} S[\Psi] \,=\, & \hat{S}[\Psi] + \frac{1}{4}\int \mathrm{d}^8z \,\Big(D\,\Psi + \bar{D}\,\bar{\Psi} \Big)^2 - \frac{1}{2}\int \mathrm{d}^8z\, \Big\{ \Psi^\alpha \bar{D}^2 \Psi_\alpha + \bar{\Psi}_{\dot\alpha} D^2 \bar{\Psi}^{\dot\alpha} \Big\} + \\ & + m \int \mathrm{d}^8z \,\Big\{ \Psi^2 + \bar{\Psi}^2 \Big\}, \end{aligned} \tag{C.8}$$

[7] At $s = 1$, the integer-superspin transverse model (A.4) provides a non-minimal off-shell realization for the massless gravitino multiplet [14] formulated in terms of an unconstrained real scalar H and Majorana γ-traceless spin-vector $\mathbf{\Psi}_a = (\Psi_{a\beta}, \bar{\Psi}_a{}^{\dot\beta})$, with $\gamma^a \mathbf{\Psi}_a = 0$.

formulated solely in terms of Ψ_α and its conjugate. This action was obtained in [28] by applying a duality transformation to $S[\Psi, H]$ (this duality transformation converts H into a chiral spinor superfield, and the latter is then integrated out). Upon the trivial rescaling

$$\Psi_\alpha \to \mathrm{i}\,\Psi_\alpha\,, \qquad m \to -m\,, \tag{C.9}$$

the action (C.8) takes the form

$$S_m[\Psi] = \hat{S}[\Psi] - \frac{1}{4}\int \mathrm{d}^8z\,\Big(D\Psi - \bar{D}\bar{\Psi}\Big)^2 + m\int \mathrm{d}^8z\,\Big\{\Psi^2 + \bar{\Psi}^2\Big\}\,. \tag{C.10}$$

Finally, the transformation

$$\Psi_\alpha \to \mathrm{e}^{\mathrm{i}\,\pi/4}\,\Psi_\alpha\,, \qquad D_\alpha \to \mathrm{e}^{\mathrm{i}\,\pi/4}\,D_\alpha \tag{C.11}$$

turns (C.10) into[8]

$$S'_m[\Psi] = \hat{S}[\Psi] + \frac{1}{4}\int \mathrm{d}^8z\,\Big(D\Psi + \bar{D}\bar{\Psi}\Big)^2 + \mathrm{i}\,m\int \mathrm{d}^8z\,\Big\{\Psi^2 - \bar{\Psi}^2\Big\}\,. \tag{C.12}$$

The action for massive gravitino multiplet (C.10), or its equivalent form (C.12), was discovered many years ago [21]. Its massless counterpart,

$$S_{\mathrm{OS}}[\Psi] = \hat{S}[\Psi] - \frac{1}{4}\int \mathrm{d}^8z\,\Big(D\Psi - \bar{D}\bar{\Psi}\Big)^2\,, \tag{C.13}$$

is the Ogievetsky-Sokatchev action for massless gravitino multiplet [47]. The massless actions (C.1) and (C.13) are related to each other by a duality transformation, see [11] for a review. As demonstrated in [46], the actions $S_{\mathrm{OS}}[\Psi]$ and $\hat{S}[\Psi]$ (the latter being a gauge fixed version of (C.1)) are the only possible off-shell realizations for massless gravitino multiplet in terms of a single spinor superfield.

By analogy with [10], let us apply a transformation

$$\Psi_\alpha \to \check{\Psi}_\alpha = \Psi_\alpha + D_\alpha U + \mathrm{i}\,\Sigma_\alpha\,, \qquad U \neq \bar{U}\,, \qquad \bar{D}_{\dot\alpha}\,\Sigma_\alpha = 0 \tag{C.14}$$

to the action $S_{\mathrm{OS}}[\Psi]$. One obtains

$$\begin{aligned} S_{\mathrm{OS}}[\check{\Psi}] \,=\, & S_{\mathrm{OS}}[\Psi] - \frac{1}{4}\int \mathrm{d}^8z\,\Big\{(U+\bar{U})\,D^\alpha\bar{D}^2D_\alpha(U+\bar{U}) - (D\Sigma + \bar{D}\bar{\Sigma})^2\Big\} + \\ & + \frac{1}{2}\int \mathrm{d}^8z\Big\{\Psi^\alpha\bar{D}^2D_\alpha(U+\bar{U}) + \mathrm{i}\,\Psi^\alpha D_\alpha(D\Sigma+\bar{D}\bar{\Sigma}) + \mathrm{c.c.}\Big\}\,. \end{aligned} \tag{C.15}$$

From here it is obvious that the Ogievetsky-Sokatchev action (C.13) possesses the following gauge freedom

$$\delta\Psi_\alpha = \mathrm{i}\,D_\alpha K_1 + \mathrm{i}\,\bar{D}^2 D_\alpha K_2\,, \qquad \bar{K}_i = K_i\,. \tag{C.16}$$

[8]In our previous publication [28], the models (C.10) and (C.12) were mistakenly treated as different, albeit similar, off-shell realizations for the massive gravitino multiplet. The transformation (C.11) establishes the equivalence of the two realizations.

Now, following [24], in (C.14) we choose

$$U = \frac{1}{2}V - \frac{\mathrm{i}}{4m}\left(D\eta + \bar{D}\bar{\eta}\right), \qquad \Sigma_\alpha = \eta_\alpha + \frac{\mathrm{i}}{4m}\bar{D}^2 D_\alpha V\,, \tag{C.17}$$

with V a real scalar, and η_α a chiral spinor. With this choice, the massive action (C.10) turns [24] into

$$\begin{aligned} S_m[\check{\Psi}] = S_{\mathrm{OS}}[\Psi] + \frac{1}{4}\int \mathrm{d}^8 z \left\{ V D^\alpha \bar{D}^2 D_\alpha V - (D\eta + \bar{D}\bar{\eta})^2 \right\} + \\ + m\int \mathrm{d}^8 z \left\{ \left(\Psi + \frac{1}{2}DV + \mathrm{i}\,\eta\right)^2 + \mathrm{c.c.} \right\}. \end{aligned} \tag{C.18}$$

In the massless limit, $m \to 0$, this action becomes a sum (with the correct signs!) of the gauge massless actions for (i) gravitino multilet; (ii) vector multiplet; (iii) tensor multiplet.

In conclusion, it is worth pointing out one final possible implication of this discussion. Many years ago [48], one of the authors (SJG) conjectured that in string field theory, there might exist a limit in which all higher spin states become massless and yet the theory retains its unitary character. Within string theory, all masses are proportional to the string tension. In the limit of no tension, all masses approach zero. Although the present example is far away from being a proof of this, the extension of the present example along the lines described by the works in [30, 31] is consistent with this conjecture.

References

[1] L.P.S. Singh and C.R. Hagen, *Lagrangian formulation for arbitrary spin, 1. The boson case*, *Phys. Rev.* **D 9** (1974) 898; *Lagrangian formulation for arbitrary spin, 2. The fermion case*, *Phys. Rev.* **D 9** (1974) 910.

[2] M. Fierz and W. Pauli, *On relativistic wave equations for particles of arbitrary spin in an electromagnetic field*, *Proc. Roy. Soc. Lond.* **A173** (1939) 211.

[3] C. Fronsdal, *Massless fields with integer spin*, *Phys. Rev.* **D 18** (1978) 3624;
J. Fang and C. Fronsdal, *Massless fields with half integral spin*, *Phys. Rev.* **D 18** (1978) 3630.

[4] C. Fronsdal, *Singletons and massless, integral spin fields on de Sitter space (elementary particles in a curved space vii)*, *Phys. Rev.* **D 20** (1979) 848;
J. Fang and C. Fronsdal, *Massless, half integer spin fields in de Sitter space*, *Phys. Rev.* **D 22** (1980) 1361.

[5] B. de Wit and D.Z. Freedman, *Systematics of higher spin gauge fields*, *Phys. Rev.* **D 21** (1980) 358.

[6] M.A. Vasiliev, *Gauge' form of description of massless fields with arbitrary spin*, *Sov. J. Nucl. Phys.* **32** (1980) 439.

[7] E.P. Likhtman, *Irreducible representations of the extension of the algebra of generators of the Poincaré group by bispinor generators*, Lebedev Physics Institute preprint No. 41, 1971.

[8] A. Salam and J.A. Strathdee, *Supergauge transformations*, *Nucl. Phys.* **B 76** (1974) 477.

[9] E. Sokatchev, *Projection operators and supplementary conditions for superfields with an arbitrary spin*, *Nucl. Phys.* **B 99** (1975) 96.

JHEP10(2005)008

[10] S.J. Gates, M.T. Grisaru, M. Roček and W. Siegel, *Superspace, or one thousand and one lessons in supersymmetry*, *Front. Phys.* **58** (1983) 1–548 [hep-th/0108200].

[11] I.L. Buchbinder and S.M. Kuzenko, *Ideas and methods of supersymmetry and supergravity or a walk through superspace*, IOP, Bristol 1998.

[12] A. Pasqua and B. Zumino, *Constraints and superspin for superPoincaré algebras in diverse dimensions*, *Phys. Rev.* **D 70** (2004) 066010 [hep-th/0404219].

[13] T. Curtright, *Massless field supermultiplets with arbitrary spin*, *Phys. Lett.* **B 85** (1979) 219.

[14] S.M. Kuzenko, A.G. Sibiryakov and V.V. Postnikov, *Massless gauge superfields of higher half integer superspins*, *JETP Lett.* **57** (1993) 534.

[15] S.M. Kuzenko and A.G. Sibiryakov, *Massless gauge superfields of higher integer superspins*, *JETP Lett.* **57** (1993) 539.

[16] W. Siegel and S.J. Gates Jr., *Superfield supergravity*, *Nucl. Phys.* **B 147** (1979) 77.

[17] S.M. Kuzenko and A.G. Sibiryakov, *Free massless higher superspin superfields on the anti-de Sitter superspace*, *Phys. Atom. Nucl.* **57** (1994) 1257.

[18] S.J. Gates Jr., S.M. Kuzenko and A.G. Sibiryakov, *$N = 2$ supersymmetry of higher superspin massless theories*, *Phys. Lett.* **B 412** (1997) 59 [hep-th/9609141].

[19] A.Y. Segal and A.G. Sibiryakov, *Explicit $N = 2$ supersymmetry for higher-spin massless fields in $D = 4$ AdS superspace*, *Int. J. Mod. Phys.* **A 17** (2002) 1207 [hep-th/9903122].

[20] S.J. Gates Jr., S.M. Kuzenko and A.G. Sibiryakov, *Towards a unified theory of massless superfields of all superspins*, *Phys. Lett.* **B 394** (1997) 343 [hep-th/9611193].

[21] V.I. Ogievetsky and E. Sokatchev, *Superfield equations of motion*, *J. Phys.* **A 10** (1977) 2021.

[22] S. Ferrara and P. van Nieuwenhuizen, *Noether coupling of massive gravitinos to $N = 1$ supergravity*, *Phys. Lett.* **B 127** (1983) 70.

[23] N. Berkovits and M.M. Leite, *First massive state of the superstring in superspace*, *Phys. Lett.* **B 415** (1997) 144 [hep-th/9709148]; *Superspace action for the first massive states of the superstring*, *Phys. Lett.* **B 454** (1999) 38 [hep-th/9812153].

[24] R. Altendorfer and J. Bagger, *Dual supersymmetry algebras from partial supersymmetry breaking*, *Phys. Lett.* **B 460** (1999) 127 [hep-th/9904213]; *Dual anti-de Sitter superalgebras from partial supersymmetry breaking*, *Phys. Rev.* **D 61** (2000) 104004 [hep-th/9908084].

[25] I.L. Buchbinder, S.J. Gates Jr., W.D. Linch and J. Phillips, *New 4D, $N = 1$ superfield theory: model of free massive superspin-3/2 multiplet*, *Phys. Lett.* **B 535** (2002) 280 [hep-th/0201096];
I.L. Buchbinder, S.J. Gates Jr., I. Linch, W.D. and J. Phillips, *Dynamical superfield theory of free massive superspin-1 multiplet*, *Phys. Lett.* **B 549** (2002) 229 [hep-th/0207243].

[26] Y.M. Zinoviev, *Massive spin-2 supermultiplets*, hep-th/0206209.

[27] T. Gregoire, M.D. Schwartz and Y. Shadmi, *Massive supergravity and deconstruction*, *JHEP* **07** (2004) 029 [hep-th/0403224].

[28] I.L. Buchbinder, J. James Gates, S., S.M. Kuzenko and J. Phillips, *Massive 4D, $N = 1$ superspin 1 and 3/2 multiplets and dualities*, *JHEP* **02** (2005) 056 [hep-th/0501199].

[29] J. Phillips, *Formulation of free higher spin supersymmetric theories in superspace*, hep-th/0502006.

[30] S.M. Klishevich and Y.M. Zinovev, *Electromagnetic interaction of a massive spin-2 particle*, *Phys. Atom. Nucl.* **61** (1998) 1527 [hep-th/9708150].

[31] Y.M. Zinoviev, *On massive high spin particles in AdS*, hep-th/0108192.

[32] R. Penrose, *Twistor quantisation and curved space-time*, *Int. J. Theor. Phys.* **1** (1968) 61;
R. Penrose and M.A.H. MacCallum, *Twistor theory: an approach to the quantization of fields and space-time*, *Phys. Rept.* **6** (1972) 241;
L.P. Hughston, *Twistors and particles*, Springer, Berlin 1979;
I.T. Todorov, *Conformal description of spinning particles*, Springer, Berlin 1986;
A.K.H. Bengtsson, I. Bengtsson, M. Cederwall and N. Linden, *Particles, superparticles and twistors*, *Phys. Rev.* **D 36** (1987) 1766;
P. Claus, M. Günaydin, R. Kallosh, J. Rahmfeld and Y. Zunger, *Supertwistors as quarks of* $\mathrm{SU}(2,2|4)$, *JHEP* **05** (1999) 019 [hep-th/9905112];
W. Siegel, *Fields*, hep-th/9912205.

[33] M.A. Vasiliev, *Higher spin gauge theories: star-product and AdS space*, in *The many faces of the superworld: Yuri Golfand memorial volume*, M.A. Shifman ed., World Scientific, Singapore 2000 [hep-th/9910096].

[34] S.I. Goldberg, *Curvature and homology*, Dover, New York 1982.

[35] S.J. Gates Jr., *Super p form gauge superfields*, *Nucl. Phys.* **B 184** (1981) 381.

[36] R. Grimm, M. Sohnius and J. Wess, *Extended supersymmetry and gauge theories*, *Nucl. Phys.* **B 133** (1978) 275;
M.F. Sohnius, *Bianchi identities for supersymmetric gauge theories*, *Nucl. Phys.* **B 136** (1978) 461.

[37] E. Witten, *An interpretation of classical Yang-Mills theory*, *Phys. Lett.* **B 77** (1978) 394.

[38] J. Isenberg, P.B. Yasskin and P.S. Green, *Nonselfdual gauge fields*, *Phys. Lett.* **B 78** (1978) 462.

[39] A.A. Rosly, *Super Yang-Mills constraints as integrability conditions*, in *Group theoretical methods in physics*, M.A. Markov ed., Nauka, Moscow 1983.

[40] A. Galperin, E. Ivanov, S. Kalitsyn, V. Ogievetsky and E. Sokatchev, *Unconstrained* $N=2$ *matter, Yang-Mills and supergravity theories in harmonic superspace*, *Class. and Quant. Grav.* **1** (1984) 469;
A.S. Galperin, E. A. Ivanov, V. I. Ogievetsky and E. S. Sokatchev, *Harmonic superspace*, Cambridge University Press, Cambridge 2001.

[41] S.J. Gates Jr. and H. Nishino, $D=2$ *superfield supergravity, local (supersymmetry)-squared and nonlinear sigma models*, *Class. and Quant. Grav.* **3** (1986) 391.

[42] W. Siegel, *Gauge spinor superfield as a scalar multiplet*, *Phys. Lett.* **B 85** (1979) 333.

[43] S.M. Kuzenko, *On massive tensor multiplets*, *JHEP* **01** (2005) 041 [hep-th/0412190].

[44] E.S. Fradkin and M.A. Vasiliev, *Minimal set of auxiliary fields and S-matrix for extended supergravity*, *Lett. Nuovo Cim.* **25** (1979) 79.

[45] B. de Wit and J.W. van Holten, *Multiplets of linearized* $\mathrm{SO}(2)$ *supergravity*, *Nucl. Phys.* **B 155** (1979) 530.

JHEP10(2005)008

[46] S.J. Gates Jr. and W. Siegel, *(3/2, 1) superfield of O(2) supergravity*, *Nucl. Phys.* **B 164** (1980) 484.

[47] V.I. Ogievetsky and E. Sokatchev, *On gauge spinor superfield*, *JETP Lett.* **23** (1976) 58.

[48] S.J. Gates Jr., *Unconstrained BRST: a new prototype for covariant string field theory?*, in *Strings '90*, M. Duff, E. Sezgin and C. Pope eds., World Scientific, Singapore 1990.